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Quarkonium at $T > 0$

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Abstract

We report recent progress on theoretical investigations of quarkonia at finite temperature. We discuss medium modification of charmonia and bottomonia from a viewpoint of local operators and point out that while charmonia are sensitive to the deconfinement transition bottomonia will be modified at much higher temperatures.

§1. Introduction

Properties of heavy quarkonia have been extensively studied since it was pointed out that they provide information on deconfinement transition of QCD.[1),2)] While expected suppressions of the quarkonia have been measured in heavy ion experiments,[3)] interpretation of these data is not so straightforward, because of not only the complexity of the collision processes but also the fact that spectral properties of the heavy quarkonia are not well understood yet. In this report, we focus on recent development of theoretical understanding of the quarkonium states at finite temperature.

§2. Theoretical development

Lattice QCD provides a unique non-perturbative first principle approach. A direct study of the quarkonium spectral function $\rho(\omega, T)$ is possible with the help of the maximum entropy method (MEM) which enables us to invert a current correlation function at imaginary time $G(\tau, T)$ via a dispersion relation.[4)]

$$G(\tau, T) = \int d\omega \frac{\cosh[\omega(\tau - 1/(2T))]}{\sinh[\omega/(2T)]} \rho(\omega, T). \tag{2·1}$$

Even in recent calculations with bigger lattice sizes,[5)] however, quantitative information seems hard to be extracted, presumably due to limited temporal lattice size at high temperature and a small number of data points. The existence of the spectral peak does not necessary mean survival of a quarkonium[6)] and substantial spectral modification does not contradict with the behavior of $G(\tau, T)$.[7)] A rather promising direction seems to determine an interquark potential $V(r, T)$ containing both real and imaginary parts for a Schödinger equation by lattice QCD.[8)] Indeed, existence of the imaginary part in the potential was pointed out by Laine et al. within a resummed perturbation theory.[9)] Recently significant progress has been made with an effective field theory framework for heavy quark bound states.[10)] Though this approach assumes a hierarchy of the energy scale which becomes complicated at finite temperature due to newly introduced medium energy scales, analytically results have been presented in some cases to give an insight of possible mechanisms of the in-medium modification of a quarkonium.[11)] These analyses indicate that a dominant in-medium effect on quarkonium at experimentally accessible temperatures could be a singlet to octet breakup process by gluons. On the other hand, an estimation of the energy density achieved in heavy ion collisions at RHIC energies and subsequent hydrodynamic evolution lead to a lifetime of the deconfined phase long enough to melt quarkonia if $\Gamma > 50$ MeV.[12)] This means that we need a theoretical estimation of the width at those temperatures, likely

in the strongly interacting regime.[13)] One of the promising approach for this purpose is relating local operators to a medium modification of a quarkonium via an operator product expansion (OPE). We will give a basic concept and a recent result below.

§3. Local operator approach for quarkonia

The interaction between a heavy quarkonium and soft gluons was first formulated by Peskin.[14)] The key concept is a separation scale, which is binding energy ϵ in the case of a heavy quarkonium. Regarding an exchange momentum k larger than ε as hard scale while the other case as soft one, we may express a matrix element via OPE as $\sum_i C_i \langle \mathcal{O}_i \rangle$ where C_i and $\langle \mathcal{O}_i \rangle$ stand for the Wilson coefficients responsible for the hard scale $k > \epsilon$ and expectation value of local operators for the soft scale, respectively (see Fig. 1). The leading order contribution is given by dimension four gluon condensate. At lower temperature than the seperation scale, one may impose all the medium effect on the change of the expectation value of the operators, which can be extracted from lattice calculations. Taking the real part of the matrix element immediately leads to a formula of a mass shift, which is the second order Stark effect in QCD, as $\Delta m_{\bar{Q}Q} = -\frac{7\pi^2}{18}\frac{a_0^2}{\epsilon}\left\langle \frac{\alpha_s}{\pi}\Delta \boldsymbol{E}^2 \right\rangle$ where a_0 is the Bohr radius of the Coulombic bound state and $\left\langle \frac{\alpha_s}{\pi}\Delta \boldsymbol{E}^2 \right\rangle$ is the temperature dependent part of the chromoelectric condensate. The expectation value of the dimension four gluon operators can be extracted from pressure p, energy density ε and effective coupling constant $\alpha_s^{\rm eff}(T)$ obtained in lattice gauge theory as

$$\left\langle \frac{\alpha_s}{\pi}\Delta \boldsymbol{E}^2 \right\rangle = \frac{2}{11 - \frac{2}{3}N_f} M_0(T) + \frac{3}{4}\frac{\alpha_s^{\rm eff}(T)}{\pi} M_2(T) \qquad (3\cdot 1)$$

where M_0 and M_2 correspond to the gluonic part of the QCD trace anomaly, $\varepsilon - 3p$ and that of enthalpy density, $\varepsilon + p$, respectively and shown in Fig. 1. The rapid change of the energy density results in an abrupt increase of $\left\langle \frac{\alpha_s}{\pi}\boldsymbol{E}^2 \right\rangle$ thus downward mass shift of a heavy quarkonium in the vicnity of the phase transition.

The above framework with separation scale ε is applicable for a deeply bounded heavy quark-antiquark system. In reality, this condition might be questionable for charmonium at high temperature. One can turn to the current correlation function $\Pi(q^2) = \int d^4x e^{iq\cdot x} \langle Tj(x)j(0) \rangle$ in terms of OPE by going to deep Euclidean region in momentum space, $q^2 = -Q^2$. Large negative q^2 enables us to compute $\Pi(q^2)$ in a perturbative manner with a pointlike current such as $j^\mu(x) = \bar{c}(x)\gamma^\mu c(x)$ and makes convergence property better than the above case. The relation to the physical quarkonia, $q^2 = m_{\bar{Q}Q}^2$, is kept through the dispersion relation. After the Borel transformation, which optimizes the dispersion relation such that the integral over the energy is dominated by the lowest resonance, the dispersion relation for the transformed

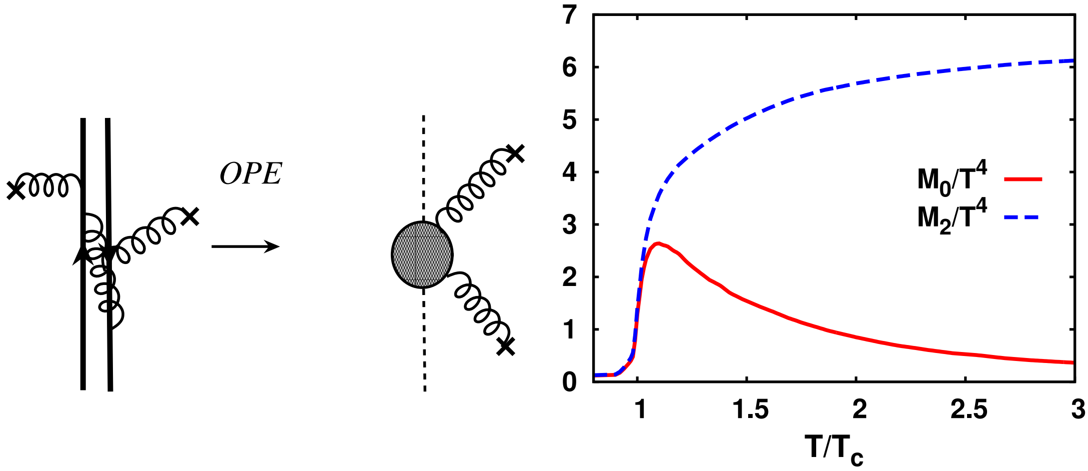

Fig. 1. Left: Schematic diagram for OPE. Right: Temperature dependence of M_0/T^4 and M_2/T^4 extracted from pure SU(3) lattice gauge theory.[15)]

correlator $M(\nu)$ reads

$$\mathcal{M}(\nu) = \int dx^2 e^{-\nu x^2} \rho(2m_Q x, T) \tag{3·2}$$

where $\nu = 4m_Q^2/M^2$ and M is the so-called Borel mass parameter. QCD sum rules[17)] gives a systematic framework to extract spectral properties from the current correlation function and dispersion relation also at finite temperature by introducing medium dependent condensates unless the typical energy scale of the medium exceeds the separation scale.[18)] With a Breit-Wigner type pole ansatz for the physical spectral function, one can derive a constraint on the spectral change of a quarkonium at finite temperature.[19),7)] Recently, Gubler and Oka proposed to apply MEM to QCD sum rules.[20)] In this method, we do not have to assume a specfic functional form on the spectral function. Futhermore, compared to lattice calculations based on Eq. (2·1), we can take as many points as possible and the dispersion relation does not have a temperature dependence other than the spectral function itself. Figure 2 shows spectral functions of J/ψ (left)[21)] and Υ (right)[22)] obtained by the QCD sum rule+MEM approach. Although resolution of the width in the lowest peak is not sufficient, one sees how the peak dissolves as temperature increases. The drastic change around T_c seen in the case of J/ψ is consistent with previous sum rule calculations,[19),7)] while Υ hardly reflects the phase transition but exhibits sizable modification above $2T_c$.

§4. Summary and outlook

Several approaches have revealed that a heavy quarkonium has a width at finite temperature induced by the medium effect, especially due to gluonic dissociations. Utilizing OPE, we have shown that charmonia are sensitive to the phase transition while bottomonium

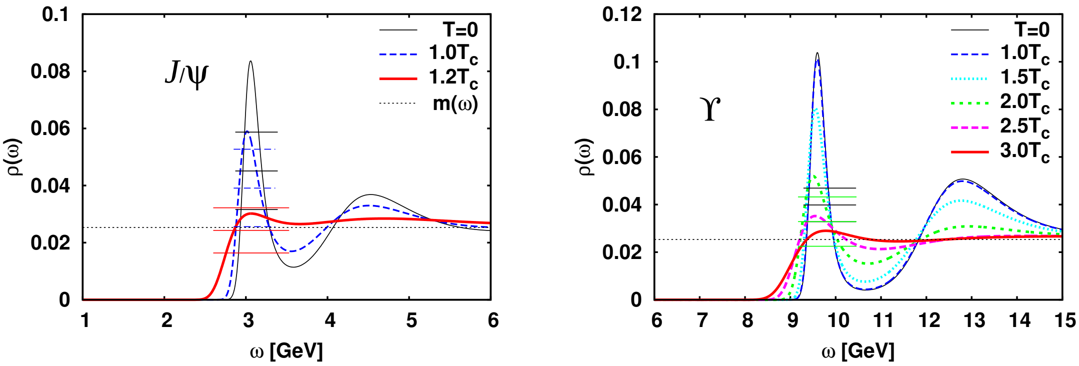

Fig. 2. Spectral function obtained from QCD sum rules with maximum entropy method. Left: J/ψ, Right: Υ.

are modified at much higher temperatures. Since bottomonia will be more appropriate for treatment with effective field theories, we expect theoretical attempts could meet together to unveil the interaction of the heavy quarkonium in the deconfined medium, which can be explored in heavy ion colllisions at the LHC energies.

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